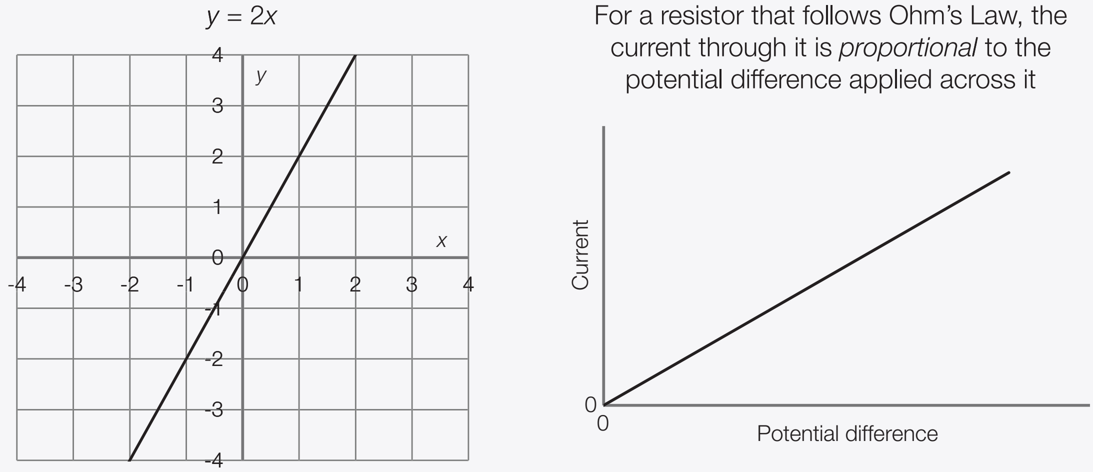
Proportional relationships

Figure 9.19 shows a ***proportional*** relationship (or a ***directly proportional*** relationship). This is a particularly common relationship in science and is discussed in detail in Chapter 5 *Working with proportionality and ratio*. The graph shows a straight line that passes through the ***origin***.

An example of this is a resistor that follows Ohm's Law, in which the current through it is proportional to the potential difference applied across it. This means that, for example, if the potential difference is doubled then the current also doubles.

Figure 9.19 Proportional relationship: $y = mx$

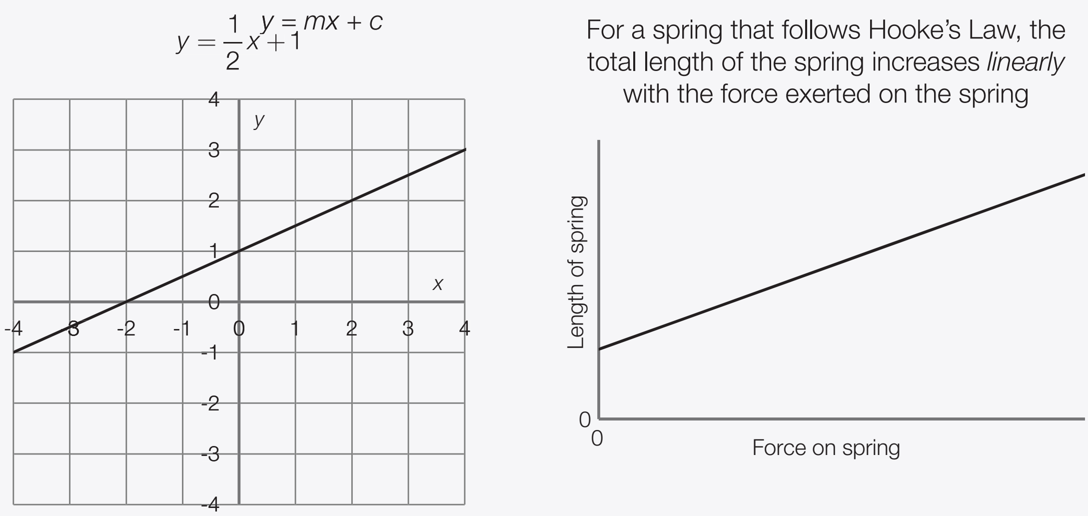


Linear relationships

Figure 9.20 shows a ***linear relationship***. This is similar to a proportional relationship in that the graph shows a straight line, but here it does not pass through the origin. An example of this is Hooke's Law, in which the total length of a spring increases linearly with the force exerted on it. This means that equal increases in force produce equal increases in the length of the spring. The ***intercept*** on the vertical axis is the length of the spring when the force on it is zero, i.e. the 'normal' length of the spring.

Note that the general equation for a proportional relationship is often written as $y = kx$, where k is the ***constant of proportionality***. In Figure 9.19 it is written as $y = mx$, in order to emphasise the similarity to the general equation for a linear relationship, $y = mx + c$, as shown in Figure 9.20. A proportional relationship is a special case of a linear relationship in which $c = 0$. Since c is the intercept on the vertical axis, this means that, when it is zero, the line passes through the origin.

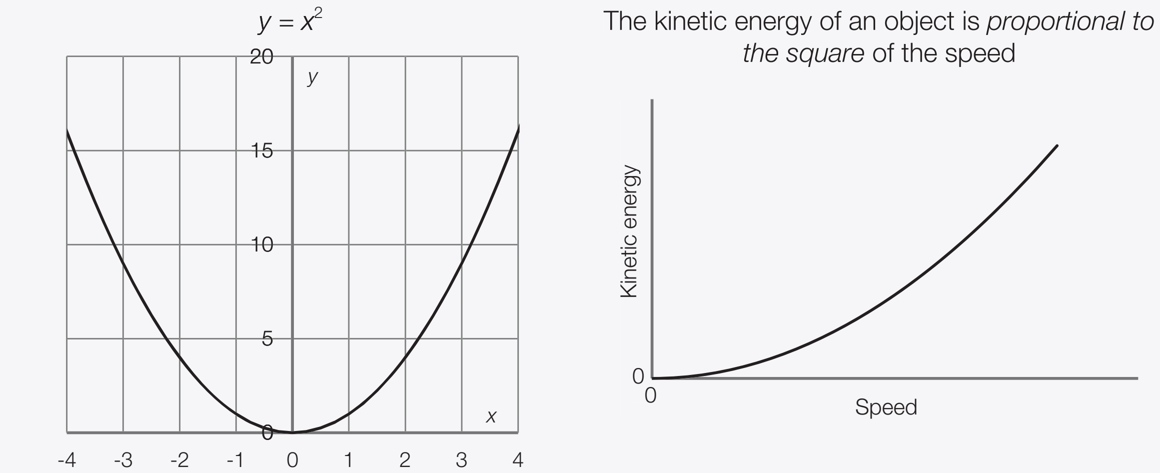
Figure 9.20 Linear relationship: $y = mx + c$



Power relationships

Figure 9.21 shows a *square* relationship. This is an example of a POWER relationship, where the independant variable is raised to a power greater than one - x^2, x^3, etc. Note here that the graph on the left includes both positive and negative values of *x*, while the science example just shows the right side of the graph representing only positive values. The example here is the relationship between the kinetic energy of an object and its speed (for which negative values would have no real-world meaning). This relationship is not linear: the line on the graph is curved, and it shows that the kinetic energy increases more rapidly than the speed.

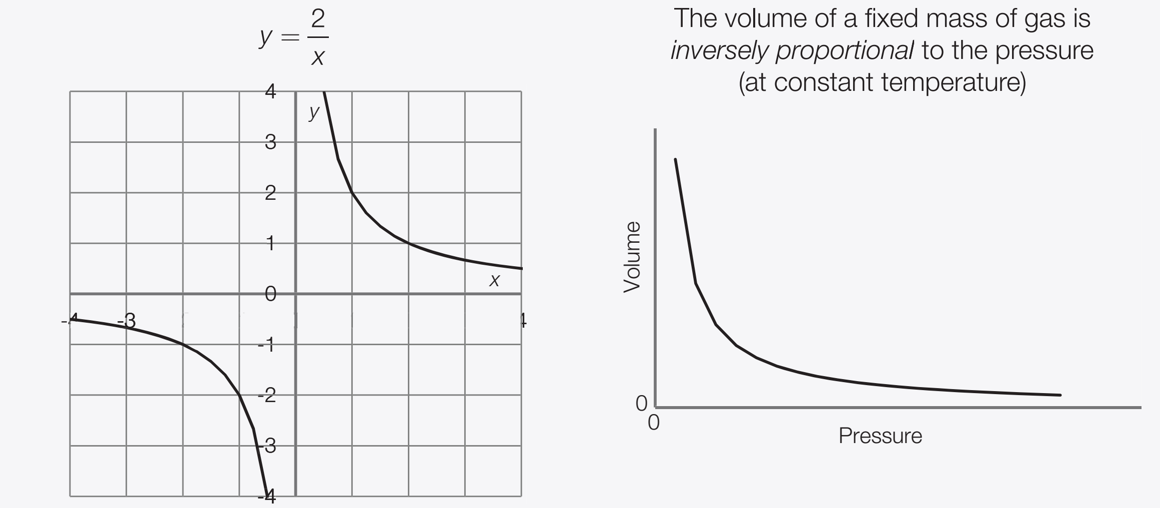
Figure 9.21 Square relationship: $y = ax^2$



Inversely proportional relationships

Figure 9.22 shows an ***inversely proportional*** relationship (or an *inverse* relationship). This kind of relationship is also discussed in Chapter 5 *Working with proportionality and ratio*. Again, note that the graph on the left shows both positive and negative values of *x*. An example in science is the relationship between the volume and pressure of a fixed mass of gas. The inverse relationship means that, for example, if the pressure is *doubled* then the volume is *halved*. Note that, as the pressure is increased, the volume gets smaller and smaller but never reaches zero (it would if the pressure were infinite but this is impossible). On the graph, therefore, the curve gets closer and closer to the horizontal axis but never actually meets it. (The technical term for the line to which a curve is tending is an *asymptote*.)

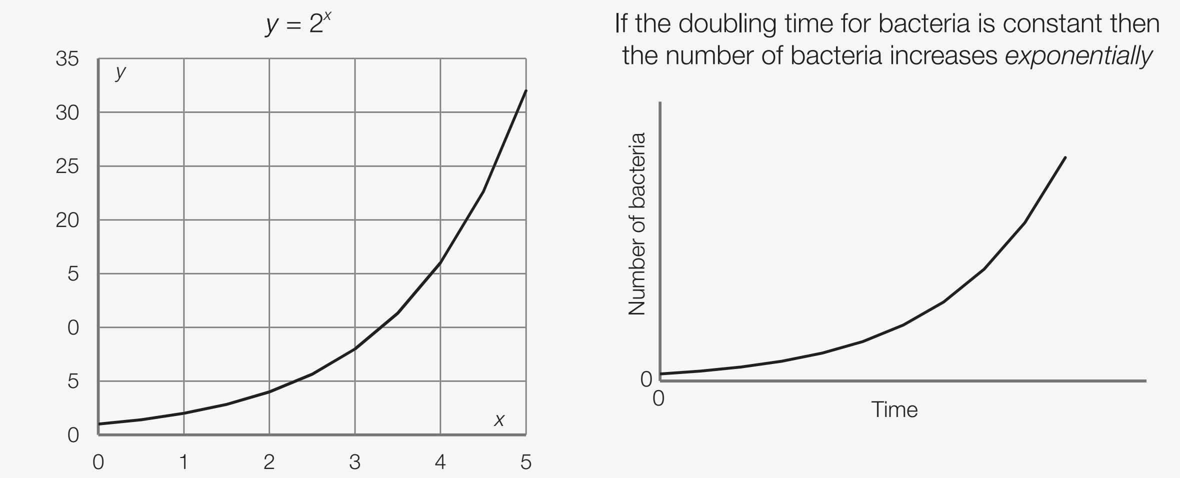
Figure 9.22 Inversely proportional relationship: $y = \frac{a}{x}$



Exponential relationships

Figure 9.23 shows an ***exponential relationship***. The rising curve on the graph looks similar to the curve for the square relationship but, in fact, an exponential curve rises much more rapidly than a square relationship does. Exponential relationships are found whenever the rate of change of a quantity is proportional to the quantity itself. For example, if the numbers of bacteria double every hour then, starting with 1 bacterium, there would be just 2 at the end of the first hour. In the fifth hour, there would be 16 at the start which would rise to 32. This leads to very rapid growth – if they continue to increase like this then there would be over 16 million at the end of 24 hours. In reality, there would be limits to the growth of increasing numbers of bacteria so, unlike the graph on the left, the curve cannot go on rising forever.

Figure 9.23 Exponential relationship: $y = a^x$



Inverse power relationships

Figure 9.24 shows an ***inverse square relationship***. This is an example where the independent variable is a numerator and raised to a power - k/x^2, or k/x^3, etc. This is similar in shape to the inverse relationship but the decrease towards the horizontal axis is rather steeper in this case. An example is the way that the intensity (or irradiance) of light from a lamp decreases as you move away from the lamp. Again, the curve approaches the horizontal axis but never meets it. So, as you move away from a lamp, the light intensity falls quite steeply but theoretically would never drop to zero, no matter how far you moved away.

Figure 9.24 Inverse square relationship: $y = \dfrac{a}{x^2}$